Chapter E
Design of Members for Compression

This chapter covers the design of compression members, the most common of which are columns. The AISC *Manual* includes design tables for the following compression member types in their most commonly available grades.

- wide-flange column shapes
- HSS
- double angles
- single angles

LRFD and ASD information is presented side-by-side for quick selection, design or verification. All of the tables account for the reduced strength of sections with slender elements.

The design and selection method for both LRFD and ASD designs is similar to that of previous AISC *Specifications*, and will provide similar designs. In this AISC *Specification*, ASD and LRFD will provide identical designs when the live load is approximately three times the dead load.

The design of built-up shapes with slender elements can be tedious and time consuming, and it is recommended that standard rolled shapes be used, when possible.

E1. GENERAL PROVISIONS

The design compressive strength, $\phi_c P_n$, and the allowable compressive strength, P_n/Ω_c, are determined as follows:

P_n = nominal compressive strength based on the controlling buckling mode

ϕ_c = 0.90 (LRFD) Ω_c = 1.67 (ASD)

Because F_{cr} is used extensively in calculations for compression members, it has been tabulated in AISC *Manual* Table 4-22 for all of the common steel yield strengths.

E2. EFFECTIVE LENGTH

In the AISC *Specification*, there is no limit on slenderness, KL/r. Per the AISC *Specification* Commentary, it is recommended that KL/r not exceed 200, as a practical limit based on professional judgment and construction economics.

Although there is no restriction on the unbraced length of columns, the tables of the AISC *Manual* are stopped at common or practical lengths for ordinary usage. For example, a double L3×3×¼, with a ⅜-in. separation has an r_y of 1.38 in. At a KL/r of 200, this strut would be 23’-0” long. This is thought to be a reasonable limit based on fabrication and handling requirements.

Throughout the AISC *Manual*, shapes that contain slender elements when supplied in their most common material grade are footnoted with the letter “c”. For example, see a W14×22[c].

E3. FLEXURAL BUCKLING OF MEMBERS WITHOUT SLENDER ELEMENTS

Nonslender sections, including nonslender built-up I-shaped columns and nonslender HSS columns, are governed by these provisions. The general design curve for critical stress versus *KL*/*r* is shown in Figure E-1.

The term *L* is used throughout this chapter to describe the length between points that are braced against lateral and/or rotational displacement.

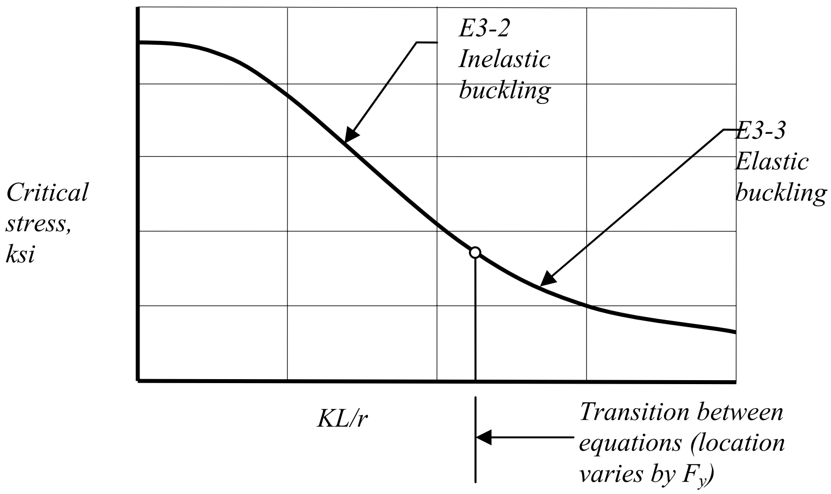

Fig. E-1. Standard column curve.

TRANSITION POINT LIMITING VALUES OF *KL*/*r*		
F_y, ksi	**Limiting *KL*/*r***	**$0.44F_y$, ksi**
36	134	15.8
50	113	22.0
60	104	26.4
70	96	30.8

E4. TORSIONAL AND FLEXURAL-TORSIONAL BUCKLING OF MEMBERS WITHOUT SLENDER ELEMENTS

This section is most commonly applicable to double angles and WT sections, which are singly-symmetric shapes subject to torsional and flexural-torsional buckling. The available strengths in axial compression of these shapes are tabulated in Part 4 of the AISC *Manual* and examples on the use of these tables have been included in this chapter for the shapes with $KL_z = KL_y$.

E5. SINGLE ANGLE COMPRESSION MEMBERS

The available strength of single angle compression members is tabulated in Part 4 of the AISC *Manual*.

E6. BUILT-UP MEMBERS

There are no tables for built-up shapes in the AISC *Manual*, due to the number of possible geometries. This section suggests the selection of built-up members without slender elements, thereby making the analysis relatively straightforward.

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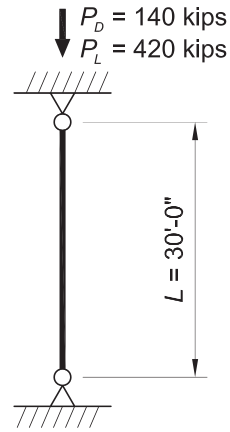
EXAMPLE E.1A W-SHAPE COLUMN DESIGN WITH PINNED ENDS

Given:

Select an ASTM A992 (F_y = 50 ksi) W-shape column to carry an axial dead load of 140 kips and live load of 420 kips. The column is 30 ft long and is pinned top and bottom in both axes. Limit the column size to a nominal 14-in. shape.



Solution:

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

LRFD	ASD
$P_u = 1.2(140\text{ kips}) + 1.6(420\text{ kips})$ $= 840\text{ kips}$	$P_a = 140\text{ kips} + 420\text{ kips}$ $= 560\text{ kips}$

Column Selection

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0.

Because the unbraced length is the same in both the x-x and y-y directions and r_x exceeds r_y for all W-shapes, y-y axis bucking will govern.

Enter the table with an effective length, KL_y, of 30 ft, and proceed across the table until reaching the least weight shape with an available strength that equals or exceeds the required strength. Select a W14×132.

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F_y = 50 ksi

Table 4-1 (continued)

Available Strength in Axial Compression, kips

W-Shapes

W14

Shape	W14×											
lb/ft	145		132		120		109		99		90	
Design	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
length, KL (ft), with respect to least radius of gyration, r_y: 0	1280	1920	1160	1750	1060	1590	958	1440	871	1310	793	1190
6	1250	1880	1130	1700	1030	1550	932	1400	848	1270	772	1160
7	1240	1860	1120	1680	1020	1530	923	1390	839	1260	764	1150
8	1230	1840	1110	1660	1010	1510	913	1370	830	1250	755	1140
9	1210	1820	1090	1640	994	1490	901	1350	819	1230	745	1120
10	1200	1800	1080	1620	980	1470	888	1340	807	1210	735	1100
11	1180	1770	1060	1600	965	1450	874	1310	794	1190	723	1090
12	1160	1750	1040	1570	948	1430	859	1290	780	1170	710	1070
13	1140	1720	1020	1540	931	1400	843	1270	766	1150	697	1050
14	1120	1690	1000	1510	912	1370	826	1240	750	1130	682	1030
15	1100	1650	982	1480	892	1340	808	1210	733	1100	667	1000
16	1080	1620	960	1440	872	1310	789	1190	716	1080	652	979
17	1060	1590	937	1410	850	1280	770	1160	698	1050	635	955
18	1030	1550	913	1370	828	1240	750	1130	680	1020	618	929
19	1010	1510	888	1330	805	1210	729	1100	661	994	601	903
20	980	1470	862	1300	782	1180	708	1060	642	964	583	877
22	927	1390	810	1220	734	1100	664	998	602	904	547	822
24	872	1310	756	1140	685	1030	620	931	561	843	509	766
26	816	1230	702	1060	635	955	574	863	519	781	472	709
28	759	1140	648	974	586	880	529	796	478	719	434	653
30	703	1060	594	893	537	807	485	729	438	658	397	597

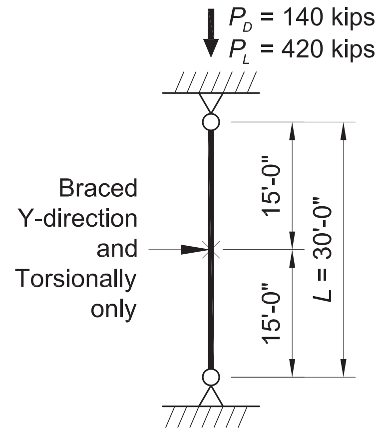
From AISC *Manual* Table 4-1, the available strength for a *y*-*y* axis effective length of 30 ft is:

LRFD	ASD
$\phi_c P_n$ = 893 kips > 840 kips **o.k.**	$\dfrac{P_n}{\Omega_c}$ = 594 kips > 560 kips **o.k.**

EXAMPLE E.1B W-SHAPE COLUMN DESIGN WITH INTERMEDIATE BRACING

Given:

Redesign the column from Example E.1A assuming the column is laterally braced about the *y-y* axis and torsionally braced at the midpoint.



Solution:

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

LRFD	ASD
P_u = 1.2(140 kips) + 1.6(420 kips) = 840 kips	P_a = 140 kips + 420 kips = 560 kips

Column Selection

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0.

Because the unbraced lengths differ in the two axes, select the member using the *y-y* axis then verify the strength in the *x-x* axis.

Enter AISC *Manual* Table 4-1 with a *y-y* axis effective length, KL_y, of 15 ft and proceed across the table until reaching a shape with an available strength that equals or exceeds the required strength. Try a W14×90. A 15 ft long W14×90 provides an available strength in the *y-y* direction of:

LRFD	ASD
$\phi_c P_n$ = 1,000 kips	$\dfrac{P_n}{\Omega_c}$ = 667 kips

The r_x/r_y ratio for this column, shown at the bottom of AISC *Manual* Table 4-1, is 1.66. The equivalent *y-y* axis effective length for strong axis buckling is computed as:

$$KL = \frac{30.0\text{ ft}}{1.66}$$
$$= 18.1\text{ ft}$$

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F_y = 50 ksi

Table 4-1 (continued)
Available Strength in Axial Compression, kips
W-Shapes

W14

Shape		W14×											
lb/ft		145		132		120		109		99		90	
Design		P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
length, *KL* (ft), with respect to least radius of gyration, r_y	0	1280	1920	1160	1750	1060	1590	958	1440	871	1310	793	1190
	6	1250	1880	1130	1700	1030	1550	932	1400	848	1270	772	1160
	7	1240	1860	1120	1680	1020	1530	923	1390	839	1260	764	1150
	8	1230	1840	1110	1660	1010	1510	913	1370	830	1250	755	1140
	9	1210	1820	1090	1640	994	1490	901	1350	819	1230	745	1120
	10	1200	1800	1080	1620	980	1470	888	1340	807	1210	735	1100
	11	1180	1770	1060	1600	965	1450	874	1310	794	1190	723	1090
	12	1160	1750	1040	1570	948	1430	859	1290	780	1170	710	1070
	13	1140	1720	1020	1540	931	1400	843	1270	766	1150	697	1050
	14	1120	1690	1000	1510	912	1370	826	1240	750	1130	682	1030
	15	1100	1650	982	1480	892	1340	808	1210	733	1100	667	1000
	16	1080	1620	960	1440	872	1310	789	1190	716	1080	652	979
	17	1060	1590	937	1410	850	1280	770	1160	698	1050	635	955
	18	1030	1550	913	1370	828	1240	750	1130	680	1020	618	929
	19	1010	1510	888	1330	805	1210	729	1100	661	994	601	903
	20	980	1470	862	1300	782	1180	708	1060	642	964	583	877
	22	927	1390	810	1220	734	1100	664	998	602	904	547	822
	24	872	1310	756	1140	685	1030	620	931	561	843	509	766
	26	816	1230	702	1060	635	955	574	863	519	781	472	709
	28	759	1140	648	974	586	880	529	796	478	719	434	653
	30	703	1060	594	893	537	807	485	729	438	658	397	597

From AISC *Manual* Table 4-1, the available strength of a W14×90 with an effective length of 18 ft is:

LRFD	ASD
$\phi_c P_n$ = 929 kips > 840 kips **o.k.**	$\dfrac{P_n}{\Omega_c}$ = 618 kips > 560 kips **o.k.**

The available compressive strength is governed by the *x-x* axis flexural buckling limit state.

The available strengths of the columns described in Examples E.1A and E.1B are easily selected directly from the AISC *Manual* Tables. The available strengths can also be verified by hand calculations, as shown in the following Examples E.1C and E.1D.

EXAMPLE E.1C W-SHAPE AVAILABLE STRENGTH CALCULATION

Given:

Calculate the available strength of a W14×132 column with unbraced lengths of 30 ft in both axes. The material properties and loads are as given in Example E.1A.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W14×132
A_g = 38.8 in.^{2}
r_x = 6.28 in.
r_y = 3.76 in.

Slenderness Check

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0.

Because the unbraced length is the same for both axes, the y-y axis will govern.

$$\frac{K_y L_y}{r_y} = \left(\frac{1.0(30.0\text{ ft})}{3.76\text{ in.}}\right)\left(\frac{12.0\text{ in}}{\text{ft}}\right)$$
$$= 95.7$$

For F_y = 50 ksi, the available critical stresses, $\phi_c F_{cr}$ and F_{cr}/Ω_c for KL/r = 95.7 are interpolated from AISC *Manual* Table 4-22 as follows:

LRFD	ASD
$\phi_c F_{cr}$ = 23.0 ksi $\phi_c P_n = 38.8\text{ in.}^2(23.0\text{ ksi})$ $= 892\text{ kips} > 840\text{ kips}$ **o.k.**	$\dfrac{F_{cr}}{\Omega_c} = 15.4\text{ ksi}$ $\dfrac{P_n}{\Omega_c} = 38.8\text{ in.}^2(15.4\text{ ksi})$ $= 598\text{ kips} > 560\text{ kips}$ **o.k.**

Note that the calculated values are approximately equal to the tabulated values.

EXAMPLE E.1D W-SHAPE AVAILABLE STRENGTH CALCULATION

Given:

Calculate the available strength of a W14×90 with a strong axis unbraced length of 30.0 ft and weak axis and torsional unbraced lengths of 15.0 ft. The material properties and loads are as given in Example E.1A.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W14×90
A_g = 26.5 in.^{2}
r_x = 6.14 in.
r_y = 3.70 in.

Slenderness Check

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0.

$$\frac{KL_x}{r_x} = \frac{1.0(30.0\text{ ft})}{6.14\text{ in.}}\left(\frac{12\text{ in.}}{\text{ft}}\right)$$
$$= 58.6 \qquad \textbf{governs}$$
$$\frac{KL_y}{r_y} = \frac{1.0(15.0\text{ ft})}{3.70\text{ in.}}\left(\frac{12\text{ in.}}{\text{ft}}\right)$$
$$= 48.6$$

Critical Stresses

The available critical stresses may be interpolated from AISC *Manual* Table 4-22 or calculated directly as follows:

Calculate the elastic critical buckling stress, F_e.

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \qquad \text{(}\textit{Spec}\text{. Eq. E3-4)}$$
$$= \frac{\pi^2(29{,}000\text{ ksi})}{(58.6)^2}$$
$$= 83.3\text{ ksi}$$

Calculate the flexural buckling stress, F_{cr}.

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}}$$
$$= 113$$

Because $\dfrac{KL}{r} = 58.6 \le 113$,

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right]F_y \qquad (\textit{Spec}.\text{ Eq. E3-2})$$
$$= \left[0.658^{\frac{50.0\text{ ksi}}{83.3\text{ ksi}}}\right]50.0\text{ ksi}$$
$$= 38.9\text{ ksi}$$

Nominal Compressive Strength

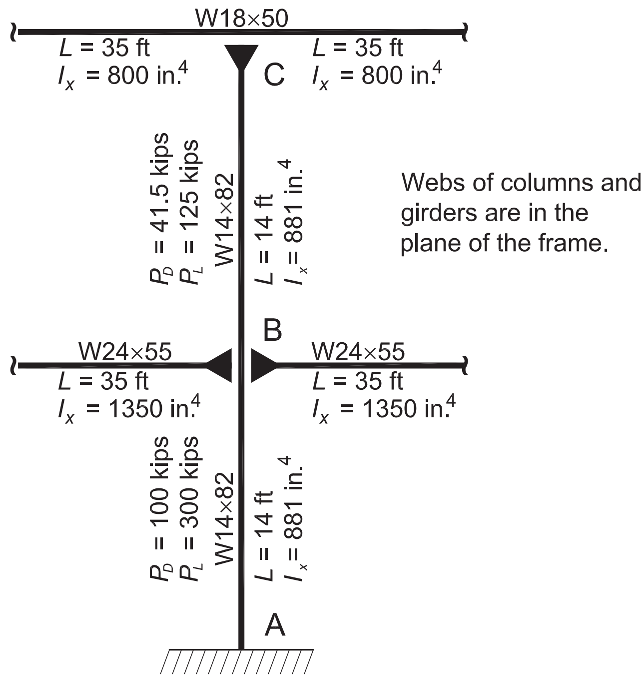
$$P_n = F_{cr}A_g \qquad (\textit{Spec}.\text{ Eq. E3-1})$$
$$= 38.9\text{ ksi}\left(26.5\text{ in.}^2\right)$$
$$= 1{,}030\text{ kips}$$

From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90(1{,}030\text{ kips})$ $= 927\text{ kips} > 840\text{ kips}$ **o.k.**	$\Omega_c = 1.67$ $\dfrac{P_n}{\Omega_c} = \dfrac{1{,}030\text{ kips}}{1.67}$ $= 617\text{ kips} > 560\text{ kips}$ **o.k.**

EXAMPLE E.4A W-SHAPE COMPRESSION MEMBER (MOMENT FRAME)

This example is primarily intended to illustrate the use of the alignment chart for sidesway uninhibited columns in conjunction with the effective length method.



Given:

The member sizes shown for the moment frame illustrated here (sidesway uninhibited in the plane of the frame) have been determined to be adequate for lateral loads. The material for both the column and the girders is ASTM A992. The loads shown at each level are the accumulated dead loads and live loads at that story. The column is fixed at the base about the *x-x* axis of the column.

Determine if the column is adequate to support the gravity loads shown. Assume the column is continuously supported in the transverse direction (the *y-y* axis of the column).

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W18×50
I_x = 800 in.^{4}

W24×55
I_x = 1,350 in.^{4}

W14×82
A_g = 24.0 in.^{2}
I_x = 881 in.^{4}

Column B-C

From Chapter 2 of ASCE/SEI 7, the required compressive strength for the column between the roof and floor is:

LRFD	ASD
$P_u = 1.2(41.5 \text{ kips}) + 1.6(125 \text{ kips})$ $= 250 \text{ kips}$	$P_a = 41.5 + 125$ $= 167 \text{ kips}$

Effective Length Factor

Calculate the stiffness reduction parameter, τ_b, using AISC *Manual* Table 4-21.

LRFD	ASD
$\dfrac{P_u}{A_g} = \dfrac{250 \text{ kips}}{24.0 \text{ in.}^2}$ $= 10.4$ ksi $\tau_b = 1.00$	$\dfrac{P_a}{A_g} = \dfrac{167 \text{ kips}}{24.0 \text{ in.}^2}$ $= 6.96$ ksi $\tau_b = 1.00$

Therefore, no reduction in stiffness for inelastic buckling will be required.

Determine G_{top} and G_{bottom}.

$$G_{top} = \tau \frac{\sum(E_c I_c / L_c)}{\sum(E_g I_g / L_g)} \qquad \text{(from \textit{Spec.} Comm. Eq. C-A-7-3)}$$

$$= (1.00)\frac{29{,}000 \text{ ksi}\left(\dfrac{881 \text{ in.}^4}{14.0 \text{ ft}}\right)}{2(29{,}000 \text{ ksi})\left(\dfrac{800 \text{ in.}^4}{35.0 \text{ ft}}\right)}$$

$$= 1.38$$

$$G_{bottom} = \tau \frac{\sum(E_c I_c / L_c)}{\sum(E_g I_g / L_g)} \qquad \text{(from \textit{Spec.} Comm. Eq. C-A-7-3)}$$

$$= (1.00)\frac{2(29{,}000 \text{ ksi})\left(\dfrac{881 \text{ in.}^4}{14.0 \text{ ft}}\right)}{2(29{,}000 \text{ ksi})\left(\dfrac{1{,}350 \text{ in.}^4}{35.0 \text{ ft}}\right)}$$

$$= 1.63$$

From the alignment chart, AISC *Specification* Commentary Figure C-A-7.2, K is slightly less than 1.5; therefore use K = 1.5. Because the column available strength tables are based on the KL about the y-y axis, the equivalent effective column length of the upper segment for use in the table is:

$$KL = \frac{(KL)_x}{\left(\dfrac{r_x}{r_y}\right)}$$

$$= \frac{1.5(14.0 \text{ ft})}{2.44}$$

$$= 8.61 \text{ ft}$$

Take the available strength of the W14×82 from AISC *Manual* Table 4-1.

At KL = 9 ft, the available strength in axial compression is:

LRFD	ASD
$\phi_c P_n = 940 \text{ kips} > 250 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 626 \text{ kips} > 167 \text{ kips}$ **o.k.**

Column A-B

From Chapter 2 of ASCE/SEI 7, the required compressive strength for the column between the floor and the foundation is:

LRFD	ASD
$P_u = 1.2(100 \text{ kips}) + 1.6(300 \text{ kips})$ $= 600 \text{ kips}$	$P_a = 100 \text{ kips} + 300 \text{ kips}$ $= 400 \text{ kips}$

Effective Length Factor

Calculate the stiffness reduction parameter, τ_b, using AISC *Manual* Table 4-21.

LRFD	ASD
$\dfrac{P_u}{A_g} = \dfrac{600 \text{ kips}}{24.0 \text{ in.}^2}$ $= 25.0 \text{ ksi}$ $\tau_b = 1.00$	$\dfrac{P_a}{A_g} = \dfrac{400 \text{ kips}}{24.0 \text{ in.}^2}$ $= 16.7 \text{ ksi}$ $\tau_b = 0.994$

Determine G_{top} and G_{bottom} accounting for column inelasticity by replacing E_cI_c with $\tau_b(E_cI_c)$. Use $\tau_b = 0.994$.

$$G_{top} = \tau \frac{\Sigma\left(E_c I_c / L_c\right)}{\Sigma\left(E_g I_g / L_g\right)}$$ (from *Spec.* Comm. Eq. C-A-7-3)

$$= \left(0.994\right)\frac{2\left(\dfrac{29{,}000 \text{ ksi}\left(881 \text{ in.}^4\right)}{14.0 \text{ ft}}\right)}{2\left(\dfrac{29{,}000 \text{ ksi}\left(1{,}350 \text{ in.}^4\right)}{35.0 \text{ ft}}\right)}$$

$$= 1.62$$

$G_{bottom} = 1.0 \text{ (fixed)}$ from AISC *Specification* Commentary Appendix 7, Section 7.2

From the alignment chart, AISC *Specification* Commentary Figure C-A-7.2, K is approximately 1.40. Because the column available strength tables are based on the KL about the y-y axis, the effective column length of the lower segment for use in the table is:

$$KL = \frac{(KL)_x}{\left(\dfrac{r_x}{r_y}\right)}$$

$$= \frac{1.40\left(14.0 \text{ ft}\right)}{2.44}$$

$$= 8.03 \text{ ft}$$

Take the available strength of the W14×82 from AISC *Manual* Table 4-1.

At KL = 9 ft, (conservative) the available strength in axial compression is:

LRFD	ASD
$\phi_c P_n = 940 \text{ kips} > 600 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 626 \text{ kips} > 400 \text{ kips}$ **o.k.**

A more accurate strength could be determined by interpolation from AISC *Manual* Table 4-1.